Lehrstuhl A für Mathematik — Aachen, den 06.12.2001
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7. Übung zu Zahlbereichserweiterungen

(Abgabe: Donnerstag, 13.12.2001, vor der Übung oder
bis 10 Uhr im Übungskasten vor dem Sekretariat des Lehrstuhls)

Aufgabe 1: Seien $a, b \in \mathbb{N}$ teilerfremd und $a \geq b$. Zeigen Sie, dass es eindeutig bestimmte $m, n \in \mathbb{N}_0$ gibt mit den Eigenschaften:

$$ma - nb = 1, \qquad m \leq b, n < a.$$ $\boxed{5}$

Aufgabe 2: Zeigen Sie:

a) Für alle $n, N \in \mathbb{N}$ mit $n < (N+1)!$ existiert eine Darstellung

$$n = \sum_{k=1}^{N} a_k k!, \qquad a_k \in \{0, 1, ..., k\}, \quad k = 1, ..., N.$$

Kann man die a_k algorithmisch beschreiben? $\boxed{4}$

b) Jedes $n \in \mathbb{N}_0$ besitzt eine eindeutige Darstellung

$$n = \sum_{k=1}^{\infty} a_k k!,$$

wobei $a_k \in \{0, 1, ..., k\}$ für alle $k \in \mathbb{N}$ und $a_k = 0$ für fast alle k. $\boxed{3}$

c) Wie groß muß man N in a) mindestens wählen um alle 100-stelligen natürlichen Zahlen darstellen zu können? $\boxed{2}$

Aufgabe 3: Ist $a = \sum_{i=0}^{n} q_i g^i$ die g-adische Darstellung von $a \in \mathbb{N}$, so heißt

$$Q_q^{'}(a) := \sum_{i=0}^{n} (-1)^i q_i$$

alternierende g-adische Quersumme von a. Zeigen Sie:

$$(g+1) \mid a \Leftrightarrow (g+1) \mid Q_q^{'}(a).$$ $\boxed{3}$

Aufgabe 4: Bestimmen Sie die g-adische Darstellung der folgenden Zahlen:

a) $1993, g = 12$, *b)* $553, g = 5$, *c)* $422, g = 8$. $\boxed{3}$

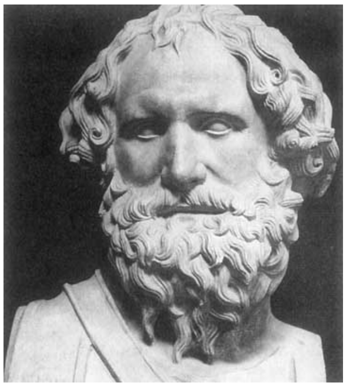
Aufgabe 5: Zeigen Sie, dass es zu jedem $a \in \mathbb{Q}$ genau ein $n \in \mathbb{Z}$ gibt mit $n \leq a < n+1$. Man nennt n das *größte Ganze* von a und schreibt $[a] := n$. $\boxed{3}$

$\boxed{\boxed{23}}$

Archimedes of Syracuse[1]

Born: 287 BC in Syracuse, Sicily
Died: 212 BC in Syracuse, Sicily





Archimedes was a native of Syracuse, Sicily. It is reported by some authors that he visited Egypt and there invented a device now known as Archimedes' screw. This is a pump, still used in many parts of the world. It is highly likely that, when he was a young man, Archimedes studied with the successors of Euclid in Alexandria.

There are quite a number of references to Archimedes in the writings of the time for he had gained a reputation in his own time which few other mathematicians of this period achieved. The reason for this was not a widespread interest in new mathematical ideas but rather that Archimedes had invented many machines which were used as engines of war. These were particularly effective in the defence of Syracuse when it was attacked by the Romans under the command of Marcellus.

Yet Archimedes, although he achieved fame by his mechanical inventions, believed that pure mathematics was the only worthy pursuit. Again Plutarch describes beautifully Archimedes attitude, yet we shall see later that Archimedes did in fact use some very practical methods to discover results from pure geometry.

The achievements of Archimedes are quite outstanding. He is considered by most historians of mathematics as one of the greatest mathematicians of all time. He perfected a methods of integration which allowed him to find areas, volumes and surface areas of many bodies. He also gave an accurate approximation to π and showed that he could approximate square roots accurately. In mechanics Archimedes discovered fundamental theorems concerning the centre of gravity of plane figures and solids. His most famous theorem gives the weight of a body immersed in a liquid, called Archimedes' principle.

His treatise *On Plane Equilibriums* sets out the fundamental principles of mechanics, using the methods of geometry. Archimedes discovered fundamental theorems concerning the centre of gravity of plane figures.

On Floating Bodies is a work in which Archimedes lays down the basic principles of hydrostatics. His most famous theorem which gives the weight of a body immersed in a liquid, called Archimedes' principle, is contained in this work.

In *Measurement of a Circle* Archimedes shows that the exact value of π lies between the values $3\frac{10}{71}$ and $3\frac{1}{7}$. This he obtained by circumscribing and inscribing a circle with regular polygons having 96 sides.

Archimedes was killed in 212 BC during the capture of Syracuse by the Romans in the Second Punic War after all his efforts to keep the Romans at bay with his machines of war had failed.

Unlike the Elements of Euclid, the works of Archimedes were not widely known in antiquity. Only after Eutocius brought out editions of some of Archimedes works, with commentaries, in the sixth century AD were the remarkable treatises to become more widely known.

[1] Aus: 'The MacTutor History of Mathematics archive' der University of St Andrews, Scotland.