Lehrstuhl A für Mathematik
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4. Übung zu Zahlbereichserweiterungen

(Abgabe: Donnerstag, 22.11.2001, vor der Übung)

Aufgabe 1: Beweisen Sie Satz 2.46 der Vorlesung. [6]

Aufgabe 2: Sei $X := \{0, 1\}$ und $X^{\mathbb{N}}$ die Menge aller Folgen mit Elementen aus X. Zeigen Sie: $X^{\mathbb{N}}$ ist überabzählbar. [4]

Aufgabe 3: Zeigen Sie, dass es unendlich viele Primzahlen der Form $4k - 1$, $k \in \mathbb{N}$, gibt. [3]

Aufgabe 4: Zeigen Sie:

a) Ist p eine Primzahl und teilt p ein Produkt $n_1 n_2 \cdots n_k$, $k \in \mathbb{N}$, natürlicher Zahlen, so teilt p auch mindestens ein n_j, $1 \leq j \leq k$. [1]

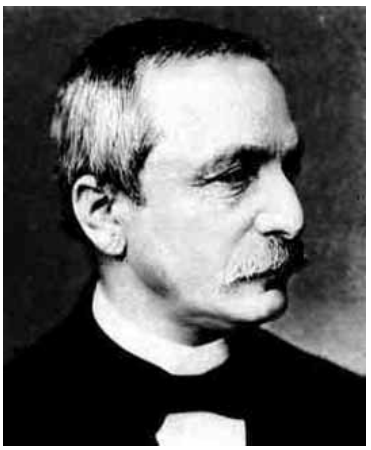
b) Eine natürliche Zahl $q \geq 2$, die mindestens einen der Faktoren m, n teilt, falls sie das Produkt $m \cdot n$ teilt, ist eine Primzahl. [3]

Aufgabe 5: Sei $n \in \mathbb{N}$, $n > 1$, mit der Eigenschaft $p \nmid n$ für jede Primzahl p mit $p^2 \leq n$. Zeigen Sie, dass n bereits eine Primzahl ist. [3]

[20]

Leopold Kronecker[1]

Born: 7 Dec 1823 in Liegnitz, Prussia (now Legnica, Poland)
Died: 29 Dec 1891 in Berlin, Germany



Kronecker's primary contributions were in the theory of equations. He made major contributions in elliptic functions and the theory of algebraic numbers.

Kronecker was taught mathematics at school by Kummer and it was due to him that Kronecker became interested in mathematics. Kronecker became a student at Berlin University in 1841 where he studied under Jacobi, Dirichlet and Eisenstein. After spending time at Bonn and Breslau he returned to Berlin to write a Ph. D. thesis on algebraic number theory under Dirichlet's supervision.

Kronecker then left Berlin and returned to Silesia where he was to make a private fortune as a banker. He remained there until 1855 when he returned to Berlin where he was to remain for the rest of his life. In 1855 Kummer came to Berlin to fill the vacancy left when Dirichlet left for Göttingen. Kronecker was not to become a professor until Kummer retired in 1883.

Kronecker was of very small stature and extremely self-conscious about his height. In fact he attacked vigorously anyone whose mathematics he disapproved.

Kronecker led the opposition to Cantor's view of set theory and the foundations of mathematics. He believed in the reduction of all mathematics to arguments involving only the integers and a finite number of steps. Kronecker is well known for his remark *God created the integers, all else is the work of man.*

Kronecker believed that mathematics should deal only with finite numbers and with a finite number of operations. The value of set theory, particularly in analysis and topology, made sure that it was accepted into mathematics despite initial opposition to the idea of the infinite.

[1] Aus: 'The MacTutor History of Mathematics archive' der University of St Andrews, Scotland.