Lehrstuhl A für Mathematik
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10. Übung zu Zahlbereichserweiterungen

(Abgabe: Donnerstag, 17.01.2002, vor der Übung oder
bis 10 Uhr im Übungskasten vor dem Sekretariat des Lehrstuhls)

Aufgabe 1: Bestimmen Sie die g-adische Darstellung der folgenden Zahlen:

a) $\frac{17}{6}$, $g = 8$, [2]

b) $\frac{6}{7}$, $g = 5$. [2]

Aufgabe 2: Bestimmen Sie alle $g \in \mathbb{N}$, $g \geq 2$ für die $\sum_{k=0}^{4} g^k$ ein Quadrat aus $\mathbb{Z}$ ist. [4]

Aufgabe 3:

a) Sei $g \in \mathbb{N}$, $g \geq 2$ und $x \in \mathbb{R}$. Zeigen Sie die Äquivalenz der beiden folgenden Aussagen:

(i) x besitzt eine endliche g-adische Darstellung, also

$$x = \varepsilon \sum_{k=-N}^{M} a_k g^{-k} \text{ mit } \varepsilon = \pm 1,\ a_k \in \{0, \dots, g-1\},\ N, M \in \mathbb{N}.$$

(ii) Es existieren teilerfremde $a, b \in \mathbb{Z}$, $b \neq 0$ und ein $m \in \mathbb{N}$ mit $x = \frac{a}{b} \in \mathbb{Q}$ und $b | g^m$. [4]

b) Für welche Grundzahlen $g \geq 2$ existiert eine endliche g-adische Darstellung aller Stammbrüche $\frac{1}{b}$, $b = 2, \dots, 12$? [2]

Aufgabe 4: Für $n \in \mathbb{N}$ sei $c_n := 1$, falls $n \in \mathbb{P}$ und $c_n := 0$, falls $n \notin \mathbb{P}$. Zeigen Sie, dass es kein $\gamma \in \mathbb{Q}$ gibt, sodass für irgendein $g \in \mathbb{N}$, $g \geq 2$ genau $\gamma = \sum_{k=1}^{\infty} c_k g^{-k}$ gilt. [3]

Aufgabe 5: Sei $g \in \mathbb{N}$, $g \geq 2$ und $(a_n)_{n \in \mathbb{N}}$ eine Abzählung von $\mathbb{Q}$, wobei die g-adische Darstellung von a_n gegeben sei durch $a_n = \varepsilon(a_n) \sum_{k=N(a_n)}^{\infty} a_{n_k} g^{-k}$. Zeigen Sie, dass dann $a := \sum_{k=1}^{\infty} a_{k_k} g^{-k} \notin \mathbb{Q}$. [3]

[20]

Augustin Louis Cauchy

(Teil 3) Political events in France meant that Cauchy was now required to swear an oath of allegiance to the new regime and when he failed to return to Paris to do so he lost all his positions there. In 1831 Cauchy went to Turin and after some time there he accepted an offer from the King of Piedmont of a chair of theoretical physics. He taught in Turin from 1832.

In 1833 Cauchy went from Turin to Prague in order to follow Charles X and to tutor his grandson. However he was not very successful in teaching the prince as this description shows:

> *When questioned by Cauchy on a problem in descriptive geometry, the prince was confused and hesitant. ... As with mathematics, the prince showed very little interest in these subjects. Cauchy became annoyed and screamed and yelled. The queen sometimes said to him, soothingly, smilingly, 'too loud, not so loud'.*

De Prony died in 1839 and his position at the Bureau des Longitudes became vacant. Cauchy was elected but, after refusing to swear the oath, was not appointed and could not attend meetings or receive a salary.

In 1843 Lacroix died and Cauchy became a candidate for his mathematics chair at the Collge de France. Liouville and Libri were also candidates. Cauchy should have easily been appointed on his mathematical abilities but his political and religious activities, such as support for the Jesuits, became crucial factors. Libri was chosen, clearly by far the weakest of the three mathematically.

When Louis Philippe was overthrown in 1848 Cauchy regained his university positions. However he did not change his views and continued to give his colleagues problems. Libri, who had been appointed in the political way described above, resigned his chair and fled from France. Liouville and Cauchy were candidates for the chair again in 1850 as they had been in 1843. After a close run election Liouville was appointed. Subsequent attempts to reverse this decision led to very bad relations between Liouville and Cauchy.

Another, rather silly, dispute this time with Duhamel clouded the last few years of Cauchy's life. This dispute was over a priority claim regarding a result on inelastic shocks. Duhamel argued with Cauchy's claim to have been the first to give the results in 1832. Poncelet referred to his own work of 1826 on the subject and Cauchy was shown to be wrong. However Cauchy was never one to admit he was wrong.

Numerous terms in mathematics bear Cauchy's name: the *Cauchy integral theorem*, in the theory of complex functions, the *Cauchy-Kovalevskaya existence theorem* for the solution of partial differential equations, the *Cauchy-Riemann equations* and **Cauchy sequences**. He produced 789 mathematics papers, an incredible achievement.