Lehrstuhl A für Mathematik
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Aachen, den 12.05.2000

4. Übung zur Vorlesung Topologie

(Abgabe: Freitag, 19.05.2000, bis 11.45 Uhr im Übungskasten)

Hinweis: Am 19. Mai 2000 fällt die Vorlesung wegen des Aachener Wissenschaftsfestes aus. Das 5. Übungsblatt wird deshalb voraussichtlich ab Donnerstag, 18. Mai 2000, auf dem Übungskasten vor dem Lehrstuhl A für Mathematik zur Mitnahme ausgelegt sein. Außerdem sind die Übungsblätter dann auch im Internet erhältlich unter

http://www.mathA.rwth-aachen.de/lehre/SS00/Topologie

Aufgabe 1: Aufgabe 1, 2. Übung.

Aufgabe 2: Aufgabe 2, 2. Übung.

Aufgabe 3: a) Zwei Metriken d und d' auf X heißen äquivalent, wenn es Konstanten $\alpha,\ \beta > 0$ gibt, so dass $\alpha\, d \leq d' \leq \beta\, d$. Zeigen Sie, dass äquivalente Metriken auf X dieselbe Topologie auf X induzieren, d. h. $\mathcal{T}_d = \mathcal{T}_{d'}$. $\boxed{2}$
b) Sei (X, d) ein metrischer Raum. Zeigen Sie:
(i) Durch $d' := \dfrac{d}{1+d}$ wird auf X eine Metrik definiert. $\boxed{3}$
(ii) d und d' induzieren dieselbe Topologie, d. h. $\mathcal{T}_d = \mathcal{T}_{d'}$. $\boxed{3}$
(iii) d und d' sind genau dann äquivalent, wenn d beschränkt ist. $\boxed{1}$

Aufgabe 4: Ein metrischer Raum (X, d) heißt ultrametrisch, falls die verschärfte Dreiecksungleichung

$$d(x,y) \leq \max\{d(x,z), d(z,y)\} \qquad \text{für alle } x,\ y,\ z \in X$$

gilt. Zeigen Sie, dass in einem ultrametrischen Raum (X, d) gilt:
a) $\forall\ x,y,z \in X \quad d(x,z) \neq d(z,y) \quad \Rightarrow \quad d(x,y) = \max\{d(x,z), d(z,y)\}$ $\boxed{2}$
b) Die (offenen) ε–Kugeln in X sind zugleich abgeschlossen. $\boxed{2}$
c) Zwei ε–Kugeln sind entweder disjunkt oder eine ist in der anderen enthalten. $\boxed{2}$

Aufgabe 5: Belegen Sie an einem Beispiel:
In einem metrischen Raum (X, d) gilt im allgemeinen für eine offene Kugel $K_r(x)$ weder $\overline{K_r(x)} = \{y \in X\ ;\ d(x,y) \leq r\}$ noch $\partial K_r(x) = \{y \in X\ ;\ d(x,y) = r\}$. $\boxed{3}$

$\boxed{\boxed{18}}$

Kazimierz Kuratowski[1]

Born: 2 Feb 1896 in Warsaw, Poland
Died: 18 June 1980 in Warsaw, Poland

Kazimierz Kuratowski's father, Marek Kuratowski was a leading lawyer in Warsaw. When Kuratowski left secondary school he decided that he wanted to become an engineer. The University of Glasgow, in Scotland, had an engineering school with a long established history, the chair of engineering being established in 1840. It rightly appeared to Kuratowski as an outstanding place to study engineering.

At the end of his first year Kuratowski was awarded the Class Prize in Mathematics. He then studied chemistry at the Technical College during the summer and returned to Poland for a holiday before starting his second year of study. However, back in Poland in August 1914 at the outbreak of World War I, returning to Scotland became impossible for Kuratowski. Although his education was disrupted, one benefit to mathematics was that Kuratowski could no longer study engineering and mathematics would gain enormously. In August 1915 the Russian forces which had held Poland for many years withdrew from Warsaw. One of the first moves after the Russian withdrawal was the refounding of the University of Warsaw and it began operating as a Polish university in November 1915. Kuratowski was one of the first students to study mathematics when the university reopened. He attended seminars given by Janiszewski and Mazurkiewicz in Warsaw before the end of the war. He writes:

As early as 1917 Janiszewski and Mazurkiewicz were conducting a topology seminar, presumably the first in that new, exuberantly developing field. The meeting of that seminar, taken up to a large extent with sometimes quite vehement discussions between Janiszewski and Mazurkiewicz, were a real intellectual treat for the participants.

There were two others on the staff at the University of Warsaw who were also to have a major influence on Kuratowski. One was Lukasiewicz, a professor of philosophy who worked on mathematical logic. The second person, who arrived in 1918, was Sierpinski. After graduating in 1919, Kuratowski undertook his doctoral studies working under Janiszewski and Mazurkiewicz. In 1921 Kuratowski was awarded his doctorate.

Kuratowski was appointed as a professor at the Technical University of Lvov in 1927. At Lvov he worked with Banach and they answered some fundamental problems on measure theory.

[1] Aus: 'The MacTutor History of Mathematics archive' der University of St Andrews, Scotland (gekrzt).

In 1934 he left Lvov and became professor of mathematics at the University of Warsaw. He was to spend the rest of his career at the University of Warsaw although he became involved in mathematical activities which saw him travelling world-wide. It was now that Kuratowski began to devote his energies to the cause of Polish mathematics rather than to give all his efforts to his research.

After the German invasion of Poland in 1939 life there became extremely difficult. There was a strategy by the invaders to put an end to the intellectual life of Poland and to achieve this they sent many academics to concentration camps and murdered others. The Poles had experience of surviving such attacks, however, and they employed the same tactics as they had during the period of Russian domination and organised an underground university in Warsaw. Kuratowski risked his life to teach in this illegal educational establishment through the war.

At the end of World War II the whole educational system was destroyed and had to be completely rebuilt. It was Kuratowski who now took on the role of leader in this rebuilding process. He was appointed the Director of the Mathematical Institute of the Polish Academy of Sciences in 1949. Despite being 53 years of age when appointed, Kuratowski held this position of director for 19 years. He held other positions of importance in the Polish scientific scene.

Kuratowski's main work was in the area of topology and set theory. He used the notion of a limit point to give closure axioms to define a topological space. In 1922 he used Boolean algebra to characterise the topology of an abstract space independently of the notion of points. Subsequent research showed that, together with Felix Hausdorff's definition of topological space in terms of neighbourhoods, the closure operator yielded more fertile results than the axiomatic theories based on Maurice Frchet's convergence (1906) and Frigyes Riesz's point of accumulation (1907).

Other major contributions by Kuratowski were to compactness and metric spaces. He was the author of *Topologie*, which was the crowning achievement of the Warsaw School in point set topology. The first volume of this work was the major source on metric spaces for several decades.

His 1930 work on non-planar graphs is of fundamental importance in graph theory, he showed that a necessary and sufficient condition for a graph G to be planar is that it does not contain a subgraph homeomorphic to either K_5 or $K_{3,3}$.

His work in set theory considered a function as a set of ordered pairs and this made the function notion as proposed by Frege, Charles Peirce and Schrder redundant. He also considered the topology of the continuum, the theory of connectivity, dimension theory, and answered measure theory questions.

Kuratowski was honoured with prizes and election to academies. He received honorary degrees from many universities.