LEHRSTUHL A FÜR MATHEMATIK
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Aachen, den 05.05.2000

3. Übung zur Vorlesung Topologie

(Abgabe: Freitag, 12.05.2000, bis 11.45 Uhr im Übungskasten)

Aufgabe 1: Sei $(X, \mathcal{T})$ ein topologischer Raum und seien A, $B \subset X$. Zeigen Sie:

a) $\operatorname{int} A$ ist offen. $\boxed{2}$

b) $\operatorname{int} A = \bigcup\limits_{A \supset G \in \mathcal{T}} G$. $\boxed{2}$

c) $\operatorname{int} A$ ist die größte offene Teilmenge von A. $\boxed{1}$

d) $\operatorname{int} A = \complement_X(\overline{\complement_X A})$. $\boxed{2}$

e) $\operatorname{int}(\operatorname{int} A) = \operatorname{int} A$. $\boxed{1}$

f) $\operatorname{int}(A \cap B) = \operatorname{int} A \cap \operatorname{int} B$. $\boxed{2}$

Aufgabe 2: Sei $(X, \mathcal{T}$ ein topologischer Raum und $A \subset X$. Zeigen Sie:

a) $\partial A = \partial(\complement_X A)$. $\boxed{2}$

b) $\partial A = \overline{A} \cap \overline{\complement_X A}$. $\boxed{2}$

c) ∂A ist abgeschlossen. $\boxed{1}$

d) $\partial A = \overline{A} \setminus \operatorname{int} A$. $\boxed{3}$

e) $\partial A = \overline{\complement_X A} \setminus \operatorname{ext} A$. $\boxed{2}$

Aufgabe 3: A, B seien Teilmengen eines topologischen Raumes $(X, \mathcal{T})$.

a) Zeigen Sie: $\partial \mathring{A} \subset \partial A$ und $\partial \overline{A} \subset \partial A$. $\boxed{3}$

b) $\partial(A \cup B) \subset \partial A \cup \partial B$
Ist $\overline{A} \cap \overline{B} = \emptyset$, so gilt Gleichheit. $\boxed{3}$

c) Sind A, B offen, so gilt:
$[(A \cap \partial B) \cup (B \cap \partial A)] \subset \partial(A \cap B) \subset [(A \cap \partial B) \cup (B \cap \partial A) \cup (\partial A \cap \partial B)]$. $\boxed{3}$

Geben Sie in a) – c) Beispiele an, in denen die Mengen paarweise verschieden sind. $\boxed{3}$

Aufgabe 4: Sei X eine Menge und $h : \mathcal{P}(X) \to \mathcal{P}(X)$ eine Abbildung mit folgenden Eigenschaften:

(1) $h(\emptyset) = \emptyset$;

(2) für alle $A \subset X$ gilt $A \subset h(A)$;

(3) für alle $A \subset X$ gilt $h(h(A)) \subset h(A)$;

(4) für alle $A, B \subset X$ gilt $h(A \cup B) = h(A) \cup h(B)$.

Zeigen Sie, daß auf X eine eindeutig bestimmte Topologie existiert, so daß $h(A) = \overline{A}$ für alle $A \subset X$ die abgeschlossene Hülle von A in dieser Topologie ist. $\boxed{4}$

Aufgabe 5: Sei R ein kommutativer Ring mit 1. Sei $\mathrm{Max}(R)$ die Menge der maximalen Ideale in R. Für jede Teilmenge $\mathcal{S} \subseteq \mathrm{Max}(R)$ sei

$$\overline{\mathcal{S}} := \{M \in \mathrm{Max}(R)\,;\, \textstyle\bigcap_{S \in \mathcal{S}} S \subseteq M\}.$$

Zeigen Sie: Es gibt genau eine Topologie auf $\mathrm{Max}(R)$, für die $\overline{\mathcal{S}}$ der Abschluß von $\mathcal{S}$ für jede Teilmenge $\mathcal{S} \subseteq \mathrm{Max}(R)$ ist. $\boxed{4}$

$\boxed{\boxed{40}}$

Augustus De Morgan[1]

Born: 27 June 1806 in Madura, Madras Presidency, India (now Madurai, Tamil Nadu, India)
Died: 18 March 1871 in London, England

Augustus De Morgan's father John was a Lieutenant-Colonel who served in India. While he was stationed there his fifth child Augustus was born. Augustus lost the sight of his right eye shortly after birth and, when seven months old, returned to England with the family. John De Morgan died when Augustus was 10 years old.

At school De Morgan did not excel and, because of his physical disability

> *... he did not join in the sports of other boys, and he was even made the victim of cruel practical jokes by some schoolfellows.*

De Morgan entered Trinity College Cambridge in 1823 at the age of 16 where he was taught by Peacock and Whewell - the three became lifelong friends. He received his BA but, because a theological test was required for the MA, something to which De Morgan strongly objected despite being a member of the Church of England, he could go no further at Cambridge being not eligible for a Fellowship without his MA.

In 1826 he returned to his home in London and entered Lincoln's Inn to study for the Bar. In 1827 (at the age of 21) he applied for the chair of mathematics in the newly founded University College London and, despite having no mathematical publications, he was appointed.

In 1828 De Morgan became the first professor of mathematics at University College. He gave his inaugural lecture On the study of mathematics . De Morgan was to resign his chair, on a matter of principle, is 1831. He was appointed to the chair again in 1836 and held it until 1866 when he was to resign for a second time, again on a matter of principle.

His book *Elements of arithmetic* (1830) was his second publication and was to see many editions.

In 1838 he defined and introduced the term 'mathematical induction' putting a process that had been used without clarity on a rigorous basis. The term first appears in De Morgan's article *Induction (Mathematics)* in the *Penny Cyclopedia.* (Over the years he was to write 712 articles for the *Penny Cyclopedia.*) The *Penny Cyclopedia* was published by the Society for the Diffusion of Useful Knowledge, set up by the same reformers who founded London University, and that Society also published a famous work by De Morgan *The Differential and Integral Calculus.*

[1] Aus: 'The MacTutor History of Mathematics archive' der University of St Andrews, Scotland.

In 1849 he published *Trigonometry and double algebra* in which he gave a geometric interpretation of complex numbers.

He recognised the purely symbolic nature of algebra and he was aware of the existence of algebras other than ordinary algebra. He introduced De Morgan's laws and his greatest contribution is as a reformer of mathematical logic.

De Morgan corresponded with Charles Babbage and gave private tuition to Lady Lovelace who, it is claimed, wrote the first computer program for Babbage.

De Morgan also corresponded with Hamilton and, like Hamilton attempted to extend double algebra to three dimension. In a letter to Hamilton, De Morgan writes of his correspondence with Hamilton and William Hamilton. He writes

> *Be it known unto you that I have discovered that you and the other Sir W. H. are reciprocal polars with respect to me (intellectually and morally, for the Scottish baronet is a polar bear, and you, I was going to say, are a polar gentleman). When I send a bit of investigation to Edinburgh, the W. H. of that ilk says I took it from him. When I send you one, you take it from me, generalise it at a glance, bestow it thus generalised upon society at large, and make me the second discoverer of a known theorem.*

In 1866 he was a co-founder of the London Mathematical Society and became its first president. De Morgan's son George, a very able mathematician, became its first secretary. In the same year De Morgan was elected a Fellow of the Royal Astronomical Society.

De Morgan was never a Fellow of the Royal Society as he refused to let his name be put forward. He also refused an honorary degree from the University of Edinburgh. He was described by Thomas Hirst thus:

> *A dry dogmatic pedant I fear is Mr De Morgan, notwithstanding his unquestioned ability.*

Macfarlane remarks that

> ... *De Morgan considered himself a Briton unattached neither English, Scottish, Welsh or Irish.*

He also says

> *He disliked the country and while his family enjoyed the seaside, and men of science were having a good time at a meeting of the British Association in the country he remained in the hot and dusty libraries of the metropolis.* ... *he had no ideas or sympathies in common with the physical philosopher. His attitude was doubtless due to his physical infirmity, which prevented him from being either an observer or an experimenter. He never voted in an election, and he never visited the House of Commons, or the Tower, or Westminster Abbey.*

De Morgan was always interested in odd numerical facts and writing in 1864 he noted that he had the distiction of being x years old in the year x^2.